# EFFECTS OF PHASE NOISE IN HEAVILY BEAM LOADED STORAGE RINGS \*

# J. M. Byrd<sup>†</sup>

Lawrence Berkeley National Laboratory, One Cyclotron Road, Berkeley, California 94720

#### Abstract

Synchrotron oscillations in a storage ring can be driven by noise in the storage ring RF system. For phase noise in the master oscillator, which falls off exponentially with frequency, we show that beam oscillations can become significant in cases of moderate to heavy beam loading. We derive the transfer function for generator phase to beam phase and calculate the beam motion using the measured phase noise in several master oscillator sources. This is compared with measurements of the beam noise made at the Advanced Light Source, a 1.5–1.9 GeV electron storage ring.

#### 1 INTRODUCTION

In the summer of 1998, scientists using the infrared synchrotron light beam line at the Advanced Light Source (ALS) reported observations of beam motion in the frequency range of 4 to 10 kHz, variable with total beam current. Our investigations linked the beam motion to energy oscillations of the beam at a point of dispersion in the lattice. These oscillations were found to be driven by phase noise in the RF master oscillator (MO), which eventually was replaced with a lower noise MO. This paper describes the model used to calculate the effect of the phase noise on the beam and compares calculations with measurements of beam motion.

The dynamics of the interaction of a radiofrequency (RF) cavity with the beam, known as the Robinson effect[1], are well understood as are the regions of stability in the interaction. Although a storage ring RF system is always configured to Robinson stable, the effect of noise in the RF system can be significant[2], particularly in cases of moderate to heavy beam loading, where the Robinson frequency shift is large enough that the beam couples to low frequency noise sources. In this paper, we determine the beam motion driven by phase noise in master oscillator in the RF source in the limits of heavy beam loading. In particular, we are concerned with energy oscillations of the beam. Calculations of the effects are compared with beam measurements made on the Advanced Light Source, a 1.5–1.9 GeV electron storage optimized for producing synchrotron radiation.

Section 2 reviews the beam-cavity interaction using the Pedersen small-signal model the find the transfer functions between the generator phase and beam phase and energy. Section 3 presents measurements of master oscillator phase noise, calculations of the beam noise using the measured

noise, and beam measurements. Conclusions are given in section 4.

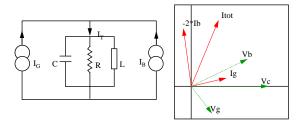


Figure 1: Phasor diagram for steady-state beam loading and the cavity equivalent circuit. The generator and beam are represented as equivalent current sources.

# 2 PEDERSEN MODEL OF BEAM-CAVITY INTERACTIONS

The Pedersen model[3, 4] is a convenient model for finding the stability of the beam-cavity interaction as well as describing the transmission of small signal deviations of the RF variables, such as the generator voltage or phase to other parameters such as the beam amplitude or phase. An equivalent circuit for the beam-cavity interaction and a phasor diagram of the relative currents and voltages is shown in Fig. 1. Transmission of small signal variations of generator, cavity, and beam amplitude and phase can be represented in a scalar signal flow diagram as shown in Fig. 2.  $a_G$ ,  $p_G$ ,  $a_G$ ,  $p_G$ ,  $a_B$  and  $p_B$ , are small amplitude variations of the generator and beam amplitude and phase, respectively. The transfer functions from total current,  $I_T$  to cavity voltage  $V_C$  are given in general by

$$G_{pp}(s) = G_{aa}(s) = \frac{1}{2} \left( \frac{Z(s+j\omega_{rf})}{Z(j\omega_{rf})} + \frac{Z(s-j\omega_{rf})}{Z(-j\omega_{rf})} \right)$$

$$G_{pa}(s) = -G_{ap}(s) = \frac{j}{2} \left( \frac{Z(s+j\omega_{rf})}{Z(j\omega_{rf})} - \frac{Z(s-j\omega_{rf})}{Z(-j\omega_{rf})} \right)$$

$$(2)$$

where the frequency  $s=j\omega$  and the subscripts refer to phase-to-phase or phase-to-amplitude modulations. Z(s) is the cavity impedance given by

$$Z(s) = \frac{2\sigma Rs}{s^2 + 2\sigma s + \omega_R^2} \tag{3}$$

R is the cavity shunt impedance,  $\omega_R$  is the cavity resonant frequency,  $\sigma = \omega_R/2Q$  is the damping rate of the cavity amplitude and Q is the quality factor. The superscript on

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<sup>†</sup> jmbyrd@lbl.gov

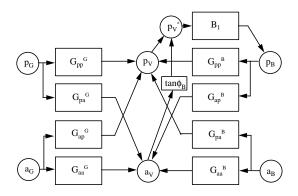


Figure 2: Scalar signal flow diagram using the Pedersen model of linear beam cavity interaction.

Parameter	Description	Value
$\overline{E}$	Beam energy	1.9 GeV
$f_{rf}$	RF frequency	499.664 MHz
$V_{rf}$	total RF voltage	1.08 MV
h	Harmonic number	328
$N_c$	Number of cells	2
R	unloaded shunt imp.	$5.3~\mathrm{M}\Omega$
Q	unloaded quality factor	43000
$\beta$	coupling beta	2.4
$\phi_L$	loading angle	-0.2 rad
$U_0$	rad. loss/turn	337 keV
$f_s$	synch. frequency	10.2 kHz
$\eta$	Momentum compaction	$1.6 \times 10^{-3}$
$ au_{rad}$	rad. damping time	13.5 msec

Table 1: Nominal ALS RF parameters.

the current to voltage transfer functions represents modulations of either the generator or beam and are given by the projections of the generator or beam current onto the total current as shown in Fig. 1.

The transfer function from generator phase modulation to beam phase modulation is given by

$$\frac{p_B}{p_C} = \frac{\left(G_{pp}^G + \tan \phi_B G_{pa}^G\right) B_1}{1 - B_1 \left(G_{pp}^B + \tan \phi_B G_{pa}^B\right)} \tag{4}$$

where  $B_1$  is the beam phase transfer function and is given by

$$B_1(s) = \frac{\omega_s^2}{s^2 + 2\sigma_B s + \omega_s^2} \tag{5}$$

where  $\sigma_B$  is the radiation damping rate. The beam energy transfer function is given by

$$\delta(s) = \frac{\omega_s s}{\omega_{rf} \alpha} B_1(s) \tag{6}$$

where  $\alpha$  is the momentum compaction.

The transfer functions for the beam phase and energy, computed for the nominal ALS beam current range are shown in Fig. 3. For example, the phase transfer function shows a downward frequency shift as well as an initial damping. These are the Robinson frequency shift and

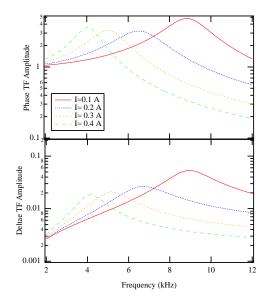


Figure 3: Computed beam phase and energy transfer functions using cavity parameters as a function of total current.

damping, respectively. To compute the total beam motion, one simply integrates the product of the noise spectrum and the transfer function. If the noise had a flat frequency distribution (i.e. white noise), the spectrum of the phase motion would be proportional to the transfer function. However, if the noise spectrum fell off exponentially at higher frequencies, as is the case with phase noise in the master oscillator, the beam would be more strongly excited at higher current with a larger Robinson frequency shift. The next section shows measurements made at the ALS confirming this.

### 3 MEASUREMENTS

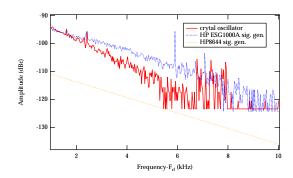


Figure 4: Phase noise from the three master oscillator sources express in dB from the carrier in a 1 Hz bandwidth. The values for the crystal oscillator and HP ESG1000A are measured and the noise for HP8644B is derived from values from the manufacturer.

The ALS presently has three MO sources: a crystal oscillator with a  $\pm 5$  kHz tuning range, an HP ESG1000A signal generator which has the feature that the frequency can be changed without interruption to the RF signal, and

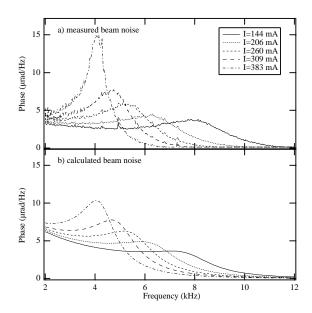


Figure 5: a) Measured beam phase noise vs. total beam current using the HP8644B signal generator as the MO. b) Calculated phase noise at the same beam currents.

an HP8644B low phase noise signal source, which was purchased when the source of the phase noise was identified. To determine the amount of beam motion driven by the phase noise in the MO, we measured the phase noise by simply observing the spectrum of phase modulation sidebands generated on a spectrum analyzer. This is shown in Fig. 4 in units of dBc per Hz from the 500 MHz carrier. Unfortunately, the noise for the HP8644B is too low to be measured in this way and thus only the catalog values are shown.

Beam phase measurements were made by recording the frequency spectrum of the sum of four beam position monitors (BPMs). The synchrotron oscillations phase modulate the beam signal, following a simple relation between the modulation sideband and the carrier. Measurements were made at a center frequency of 1499 MHz ( $3*f_{RF}$ ) to be more sensitive to phase oscillations. Shown in Fig. 5 are measurements of the spontaneous beam motion over the nominal range of beam current using the HP8644B as MO. The calculated motion shows fairly good agreement with the measured motion, particularly at lower beam current. The disagreement at high current may be due to the response of the coupled–bunch feedback system which has been neglected in calculations of the transfer function.

The effect of reducing the MO phase noise is shown in Fig. 6 which shows the spectrum of beam motion using the HP ESG1000A and HP8644B. The beam motion is reduced by the ratio of the noise in the two MOs.

## 4 CONCLUSIONS

We have identified phase noise in the 500 MHz master oscillator as a source of driven synchrotron oscillations in

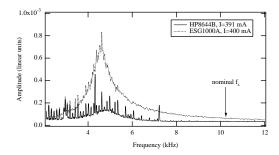


Figure 6: Measured beam noise using the two HP synthesizers described above. The reduction corresponds to the expected reduction in phase noise. Harmonics of the klystron power supply are visible with the reduced phase noise.

the ALS. The combination of the Robinson effect and the phase noise increases the coupling of the phase noise to the beam for increasing beam loading. Calculations of the beam motion from the measured MO phase noise show fairly good agreement with beam measurements. This effect can be serious for heavily loaded electron storage rings, necessitating consideration of the master oscillator design or possibly RF feedback to reduce the problem.

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